Rover Publications
United International Journal of Engineering and Sciences (UIJES)
An International Peer-Reviewed (Refereed)Engineering and Science Journal
Impact Factor: 7.984 (SJIF) Vol-4, Issue- 4,2024 ISSN:2582-5887 www.uijes.com

A Comparative Study of Hyperelastic Material Models for Polyurethane Foam

Samuel Ambosta
USA

Article Received: 10/10/2024
Article Accepted: 24/11/2024
Published Online: 27/11/2024
DOI:10.53414/UIJES.2024.4.4.17

Abstract:
Foams are widely utilized across various industries due to their unique structural properties, particularly their ability to absorb energy effectively. The behavior of these cellular solids, which can have either open or closed cell structures, plays a crucial role in applications like cushioning, padding, and protective packaging. Modeling foam behavior is essential for optimizing its application, requiring an in-depth understanding of foam flow characteristics and rheological properties. This study focuses on the compressive stress-strain behavior of polyurethane soft foam, emphasizing the typical stages of deformation—linear elasticity, plateau, and densification. Calibration data from Rogers Corporation was employed to analyze foam performance using five hyperelastic material models: Hyperfoam, Arruda-Boyce, Neo-Hookean, Ogden, and Polynomial models. Each model's accuracy was evaluated based on its ability to replicate experimental stress- strain behavior. Among these, the Arruda-Boyce model demonstrated the highest fidelity with an error of 8.6%, making it the most suitable for predicting foam behavior in finite element simulations. This work provides essential insights into material modeling for foam applications, aiding in the selection of appropriate models for diverse industrial uses.

Keywords: Foams, Hyperelastic Material Models, Finite Element Analysis, Stress-Strain, Hyperfoam, Arruda-Boyce, Neo-Hookean, Ogden, Polynomial

I. Introduction

Foams are versatile materials with a wide range of applications across various industries. These cellular solids are composed of interconnected networks of solid struts or plates that form the edges and faces of cells, which can be either open (like a sponge) or closed (like flotation foam). The unique structure of foams gives them excellent energy absorption properties, making them particularly useful in applications such as cushions, padding, and packaging materials.

Modeling foam behavior is crucial for optimizing its use in various industries and applications. Understanding the complex flow behavior and rheological properties of foams is essential for improving processes and products that involve foam materials. Accurate modeling helps predict foam behavior under different conditions, which is vital for designing and optimizing foam-based products and processes.

II. Typical Compressive Stress-Strain Curve of Foams

The compressive stress-strain behavior of foams typically exhibits three distinct stages:

1. Linear Elastic Deformation: At small strains (up to about 5%), the foam deforms in a linear elastic manner due to cell wall bending.
2. Plateau Region: Characterized by deformation at almost constant stress, caused by the elastic buckling of the cell edges or walls. In closed-cell foams, the enclosed gas pressure and membrane stretching increase the level and slope of the plateau.
3. **Densification**: In this final stage, the cell walls crush together, resulting in a rapid increase of compressive stress. Ultimate compressive nominal strains of 0.7 to 0.9 are typical for foams.

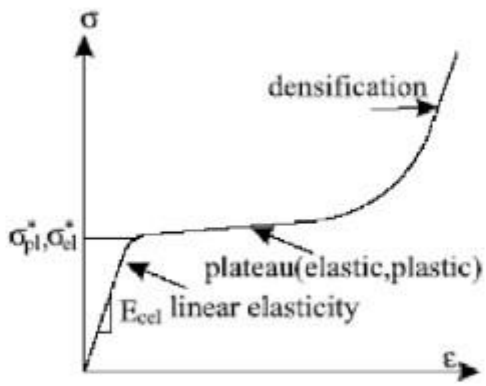

Figure 1: Typical compressive stress-strain curve for foam, illustrating the stages of linear elasticity, plateau, and densification

III. Vendor Data for Calibration

For this study, vendor data from Rogers Corporation was used for calibration. The specific product used was a polyurethane soft foam, product 30-15188, with a thickness of 4.78mm.

Testing Methods used by the Vendor:

- Metric Units
- Rate: 5.08 mm / min
- Area: 1135.5 mm²
- Form: 38 mm diameter
- Material layered to 6.35 mm thickness for testing

The vendor uses Compression Force Deflection (CFD) to measure the firmness of its foam products. CFD compresses the entirety of a material sample (generally about 10 cm) and records the amount of force (stress) at different levels of compression strain. This method provides a more accurate representation of foam firmness at different compression levels compared to durometer measurements.

Rover Publications
United International Journal of Engineering and Sciences (UIJES)
An International Peer-Reviewed (Refereed)Engineering and Science Journal
Impact Factor: 7.984 (SJIF) Vol-4, Issue- 4,2024 ISSN:2582-5887 www.uijes.com

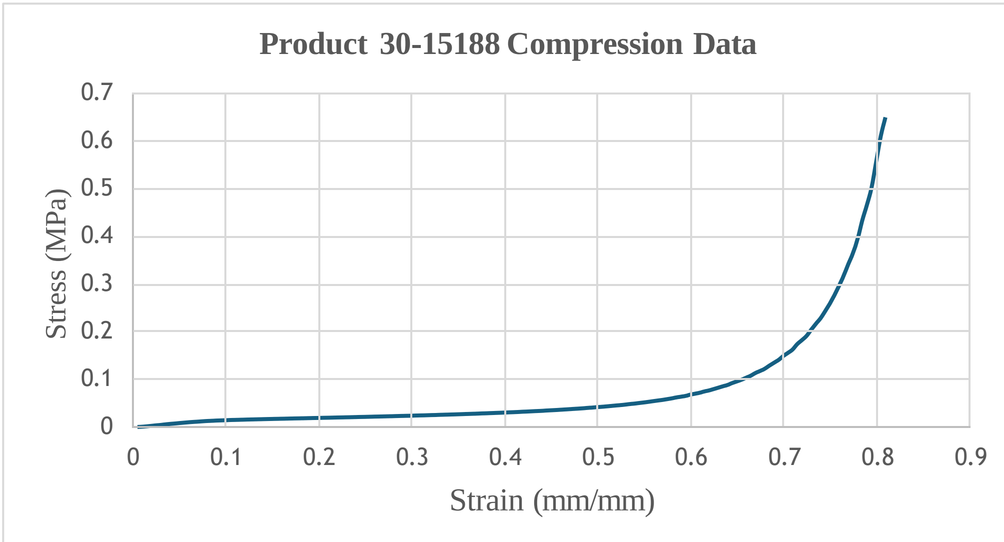

Figure 2: Compression Force Deflection (CFD) test setup and results as per Rogers Corporation data

IV. Material models

For finite element analysis (FEA) modeling, it is crucial to capture the stress-strain behavior of the foam as accurately as possible. Several hyperelastic material models are commonly used to describe the behavior of foam materials.

A. Hyperfoam Model

The hyperfoam model is based on the following strain energy function:

$$U=\sum_{i=1}^{N}\frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_3^{\alpha_i}-3+\frac{1}{\beta_i}\left(J^{-\alpha_i\beta_i}-1\right)\right)$$

Where:

- N is a material parameter,
- μi, αi_ and βi are temperature-dependent material parameters,
- λi are the principal stretches, and
- J is the total volume ratio.

B. Arruda-Boyce Model

The Arruda-Boyce model is based on the statistical mechanics of a material with a cubic representative volume element containing eight chains along the diagonal directions.

Rover Publications
United International Journal of Engineering and Sciences (UIJES)
An International Peer-Reviewed (Refereed)Engineering and Science Journal
Impact Factor: 7.984 (SJIF) Vol-4, Issue- 4,2024 ISSN:2582-5887 www.uijes.com

The strain energy density function for the incompressible Arruda-Boyce model is given by:

$$W = Nk_B\theta\sqrt{n}\,[\beta\lambda_{\text{chain}} - \sqrt{n}\ln(\frac{\sinh\beta}{\beta})]$$

Where:

- N is the number of chain segments,
- k_B is Boltzmann constant,
- θ is temperature,
- N is a constant term,
- β is a parameter related to temperature or deformation, and
- λ_{chain} is chain stretch ratio.

C. Neo-Hookean Model

The Neo-Hookean model is a simple hyperelastic material model that can be used for predicting the nonlinear stress-strain behavior of materials undergoing large deformations.

The strain energy density function for an incompressible Neo-Hookean solid is:

$$W = C_1(I_1 - 3)$$

Where:

- C_1 is a material constant related to the shear modulus
- I_1 is the first invariant of the left Cauchy-Green deformation tensor

D. Ogden Model

The Ogden material model is a hyperelastic model used to describe the non-linear stress-strain behavior of complex materials such as rubbers, polymers, and biological tissue.

The strain energy density function for the Ogden model is:

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3)$$

Where:

- μ_i and α_i are material constants,
- λi are the principal stretches, and
- N is the order of the model

E. Polynomial Model

Rover Publications
United International Journal of Engineering and Sciences (UIJES)
An International Peer-Reviewed (Refereed)Engineering and Science Journal
Impact Factor: 7.984 (SJIF) Vol-4, Issue- 4,2024 ISSN:2582-5887 www.uijes.com

The polynomial hyperelastic material model is a phenomenological model of rubber elasticity. The strain energy density function for the polynomial model is:

$$W = \sum_{i+j=1}^{N} C_{ij} (I_1 - 3)^i (I_2 - 3)^j + \sum_{i=1}^{N} \frac{1}{D_i} (J - 1)^{2i}$$

Where:

- C_{ij} and D_i are material constants
- I_1 and I_2 are the first and second invariants of the deviatoric strain tensor
- J is the determinant of the deformation gradient tensor

V. Material model calibration

A. Calibration using Hyperfoam Model

Constants	Value
μ	0.0114411
α	4.92061
nu1	0.4

Table 1: Constant values for Hyperfoam material model

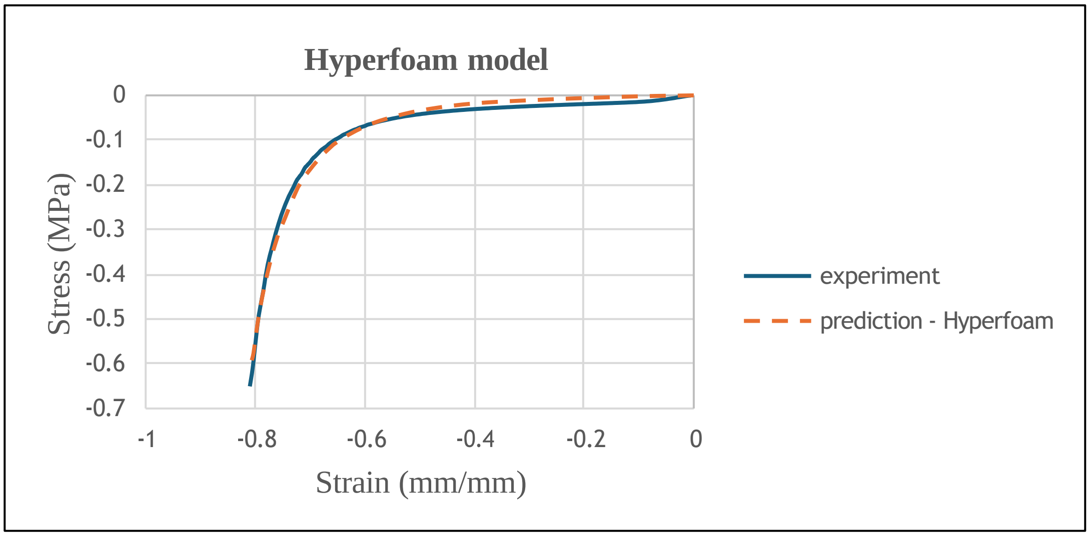

Figure 3: Experimental data calibration using Hyperfoam Model

B. Calibration using Aruda Boyce Model

Constants	Value
μ	0.00774867
λ	1.86251
D	1.25465

Table 2: Constant values for Aruda Boyce material model

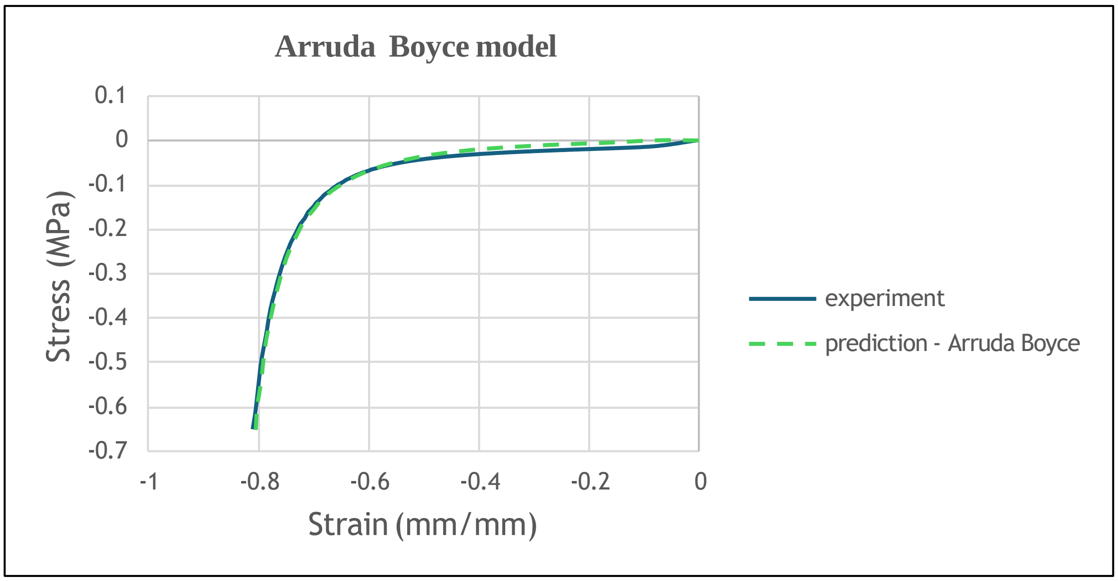

Figure 4: Experimental data calibration using Aruda Boyce material model

C. Calibration using Neo Hookean Model

Constants	Value
C10	0.0082875
D1	24.2675

Table 3: Constant values for Neo Hookean material model

Rover Publications
United International Journal of Engineering and Sciences (UIJES)
An International Peer-Reviewed (Refereed)Engineering and Science Journal
Impact Factor: 7.984 (SJIF) Vol-4, Issue- 4,2024 ISSN:2582-5887 www.uijes.com

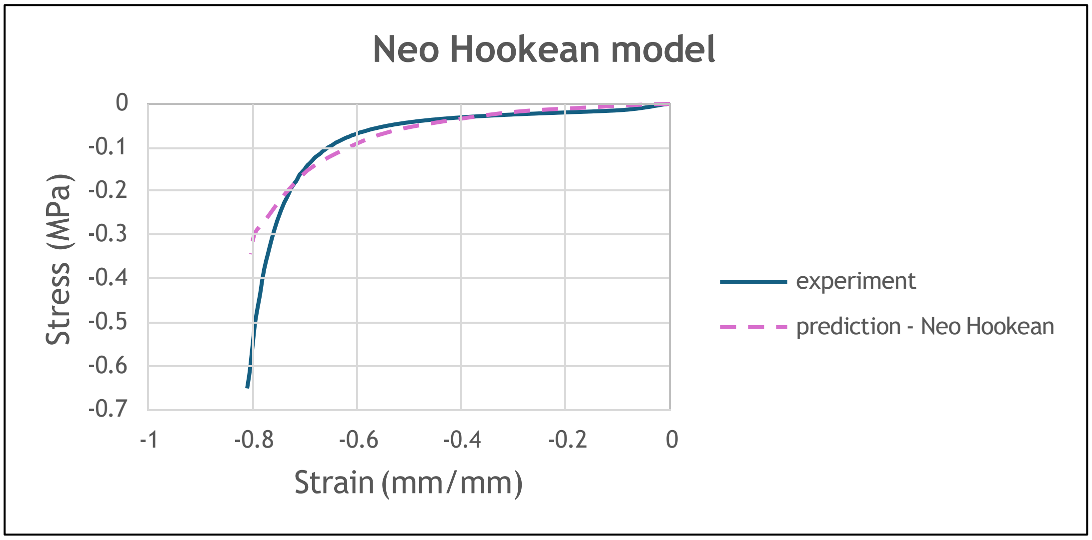

Figure 5: Experimental data calibration using Neo Hookean material model.

D. Calibration using Ogden Model

Constants	Value
C10	0.00493676
α_i	-1.90458
D1	-1.90458

Table 4: Constant values for Ogden material model.

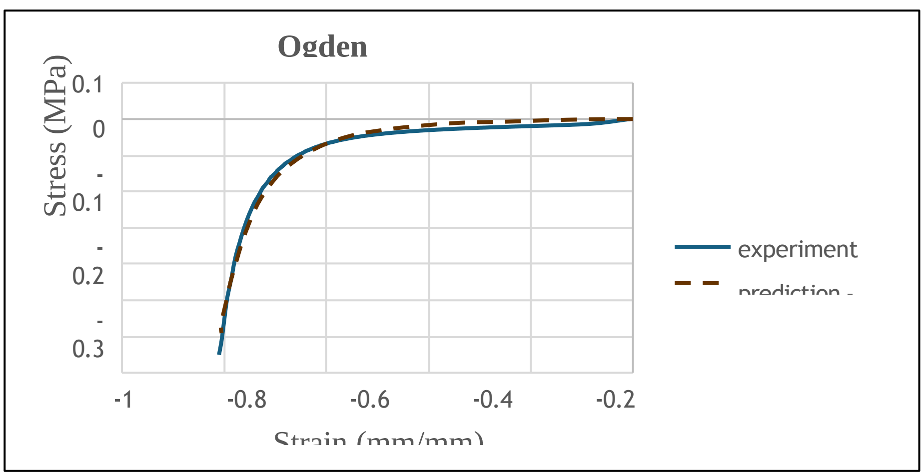

Figure 6: Experimental data calibration using Ogden material model.

E. Calibration using Polynomial Model

Rover Publications
United International Journal of Engineering and Sciences (UIJES)
An International Peer-Reviewed (Refereed)Engineering and Science Journal
Impact Factor: 7.984 (SJIF) Vol-4, Issue- 4,2024 ISSN:2582-5887 www.uijes.com

Constants	Value
C10	0.00493676
α_i	-1.90458
D1	-1.90458

Table 5: Constant values for Polynomial material model.

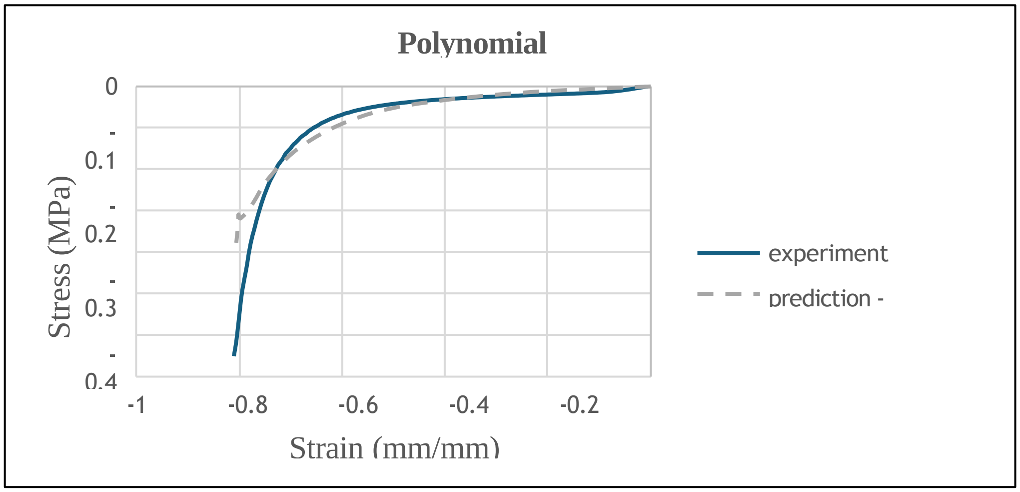

Figure 7: Experimental data calibration using Polynomial material model.

VI. Results and Discussion

This section describes the results of the different hyperelastic models used to calibrate the foam material. Table 6 shows the percentage error of the different material models used to calibrate.

Model	Error (%)
Hyperfoam	12.2
Arruda Boyce	13.9
Neo Hookean	8.6
Ogden	14.3
Polynomial	24.9

Table 6: Accuracy of the material models

Based on these results, the Arruda-Boyce model appears to provide the best fit to the experimental data, with the lowest error of 8.6%. The Hyperfoam and Neo-Hookean models also show reasonably good fits, while the Ogden and Polynomial models demonstrate higher errors in capturing the foam behavior.

These calibrated material models can be used in finite element simulations to accurately represent the behavior of the polyurethane soft foam under various loading conditions. The choice of model may depend on the specific application and the range of deformations expected in the analysis.

VII: Conclusion

This study presents a comparative analysis of five hyperelastic material models used to characterize the behavior of polyurethane soft foam. Through detailed calibration with empirical data provided by Rogers Corporation, each model's ability to predict the foam's compressive stress-strain behavior was rigorously evaluated. The findings revealed that the Arruda-Boyce model demonstrated superior accuracy, with an error margin of only 8.6%, making it the most reliable choice for simulating foam behavior in finite element analyses. The Hyperfoam and Neo-Hookean models also exhibited reasonable accuracy, while the Ogden and Polynomial models were less effective in capturing the foam's behavior during mechanical loading.

This insight underscores the importance of selecting the appropriate hyperelastic model based on the specific application and loading conditions anticipated in practical scenarios. Future research could focus on the application of these models in various dynamic loading situations, as well as the exploration of hybrid models that may combine the strengths of the existing hyperelastic frameworks while mitigating their respective limitations. Overall, the findings contribute to a deeper understanding of foam material modeling and its significance in optimizing the performance of foam-based products across diverse industrial applications.

References

[1] Neethling, S. J. (2001). *The Physical Modeling of Foam and Froth Behavior.* Wiley Online Library. https://onlinelibrary.wiley.com/doi/abs/10.1002/1521-4125%28200112%2924%3A12%3C1309%3A%3AAID-CEAT1309%3E3.0.CO%3B2-S

[2] Sadighi, M., & Salami, S. (2012). *An Investigation on Low-Velocity Impact Response of Elastomeric & Crushable Foams.* Central European Journal of Engineering, https://doi.org/10.2478/s13531-012-0026-0

[3] Edrisi, A. R. (2013). *Experimental and Modeling Study of Foam Flow in Pipes with Two Foam Flow Regimes.* Louisiana State University. https://repository.lsu.edu/gradschool_dissertations/1628/

[4] Reilly Foam Corporation. *Top Uses for Foam Products.* https://reillyfoam.com/top-uses-for-foam-products/

[5] YouTube. *Understanding Foam Behavior and Related Material Models in Abaqus.* https://www.youtube.com/watch?v=7Kkyobg7_t0

[6] Rogers Corporation. *Durometer vs. Compression Force Deflection: Which is Better?* https://www.rogerscorp.com/-/media/project/rogerscorp/documents/elastomeric-material-solutions/general/english/application-notes/durometer-vs-compression-force-deflection---which-is-better.pdf

[7] Rogers Corporation. *Testing and Evaluation Process.* https://www.rogerscorp.com/advanced-electronics-solutions/technical-expertise/testing-and-

evaluation-process
[8] Parafix. *Rogers 4701-30-15188-04 | Foams & Rubbers. https://parafix.com/product/rogers-4701-30-15188-04/
[9] Washington University in St. Louis. *17.5.2 Hyperelastic Behavior in Elastomeric Foams. https://classes.engineering.wustl.edu/2009/spring/mase5513/abaqus/docs/v6.6/books/usb/pt05ch17s05abm08.html
[10] Wikipedia. *Arruda–Boyce Model. https://en.wikipedia.org/wiki/Arruda%E2%80%93Boyce_model
[11] Wikipedia. *Neo-Hookean Solid. https://en.wikipedia.org/wiki/Neo-Hookean_solid
[12] Wikipedia. *Ogden Hyperelastic Model. https://en.wikipedia.org/wiki/Ogden_hyperelastic_model
[13] Wikipedia. *Polynomial Hyperelastic Model. https://en.wikipedia.org/wiki/Polynomial_hyperelastic_model
[14] Dassault Systèmes Simulia Corp. *4.6.1 Hyperelastic Material Behavior - ABAQUS Theory Manual (Version 6.6).* Washington University in St. Louis. https://classes.engineering.wustl.edu/2009/spring/mase5513/abaqus/docs/v6.6/books/stm/ch04s06ath123.html